Black Hole Algorithm for Solving Optimal Reactive Power Dispatch Problem

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Abstract— This paper proposes a new heuristic algorithm that is enthused by the black hole phenomenon called as Black Hole Algorithm (BHA) used for solving the multi-objective reactive power dispatch problem. The Black Hole Algorithm (BHA) starts with a preliminary population of contestant solutions to an optimization problem. For all iteration of the black hole algorithm, the most excellent candidate is preferred to be the black hole, which followed by pulling further candidates around it, called stars. If a star move very close to the black hole, it will be swallowed by the black hole and is disappeared everlasting. In such a case, a fresh star - candidate solution is arbitrarily generated and positioned in the search space and starts a fresh search. In order to evaluate the efficiency of the proposed algorithm; it has been tested on standard IEEE 30 bus system and compared to other specified algorithms. Simulation results show that BHA is superior to other algorithms in reducing the real power loss and improving the voltage stability.

Keywords— Black hole algorithm, optimal reactive power, transmission loss

I. INTRODUCTION

The Optimal power flow problem, which was projected by Carpenter about 50 years ago and it, is one of the main problem in power-systems operation and control. Power system operation and control problem can be divided into two subordinate problems, i.e. optimal reactive power dispatch (ORPD) problem and optimal real power dispatch problem. ORPD in power systems is fretful with the security and economy of the power system operation. The reactive power dispatch is associated to the allotment of reactive power generation, which is used to minimize real power transmission losses and to maintain voltage profiles within the limits. The gradient method [1, 2], Newton method [3] and linear programming [4-7] experience from the intricacy of managing the inequality constraints. In recent times well-known Optimization techniques such as genetic algorithms have been proposed to solve the reactive power flow problem [8, 9]. In recent years, the complicatedness of voltage stability and voltage fall down has become most vital concern in power system expansion. This paper presents the reactive power dispatch problem as multi-objective optimization problem with real power loss minimization and maximization of static voltage stability margin (SVSM). Voltage stability evaluation is done by using modal analysis [10] and it is used as the pointer of voltage stability. Various evolutionary algorithms like evolutionary programming [11], PSO using multi-agent [12], cooperative co-evolutionary differential evolution [13], differential evolution [14], learning particle swarm optimization [15], self-adaptive real coded genetic algorithm [16], were developed to solve the ORPD problem. In this paper, BHA utilized for solving ORPD problem. The fundamental design of a black hole is basically an area of space that has so much mass concerted in it and there is no means for a close by object to get away from the gravitational heave. The projected black hole algorithm (BHA) begins with a preliminary population of candidate solutions to an optimization problem. In this paper Black Hole Algorithm (BHA) [17] used to solve the optimal reactive power problem. The performance of BHA has been evaluated in standard IEEE 30 bus test system and the simulation results shows that proposed method outperforms all approaches investigated in this paper.

II. VOLTAGE STABILITY EVALUATION

A. Modal Analysis for Voltage Stability Evaluation

The linearized steady state system power flow equations are given by,

\[
\begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix} =
\begin{bmatrix}
J_{p0} & J_{pv} \\
J_{q0} & J_{qv}
\end{bmatrix}
\begin{bmatrix}
\Delta V \\
\Delta \delta
\end{bmatrix}
\]

(1)

Where

\[
\Delta P = \text{Incremental change in bus real power.} \\
\Delta Q = \text{Incremental change in bus reactive power injection.} \\
\Delta \delta = \text{incremental change in bus voltage angle.} \\
\Delta V = \text{Incremental change in bus voltage Magnitude}
\]

\[J_{p0}, J_{pv}, J_{q0}, J_{qv}\] jacobian matrix are the sub-matrixes of the System voltage stability is affected by both P and Q. However at each operational point we keep P constant and evaluate voltage stability by considering incremental relationship between Q and V.
To reduce (1), let \( \Delta P = 0 \), then.
\[
\Delta Q = [I_{QV} - I_{Q1}p_{0}^{-1}] \Delta V = I_{R} \Delta V
\]
\[
\Delta V = J^{-1} \Delta Q
\]
Where
\[
I_{R} = (I_{QV} - I_{Q1}p_{0}^{-1}) P_{V}
\]
\( I_{R} \) is called the reduced Jacobian matrix of the system.

B. Modes of Voltage Instability

Voltage Stability characteristics are computed by the Eigen values and Eigen vectors.

Let
\[
J_{R} = \xi^{*} \Lambda \eta
\]
Where,
\[
\xi = \text{right eigenvector matrix of } J_{R}
\]
\( \eta = \text{left eigenvector matrix of } J_{R} \)
\( \Lambda = \text{diagonal Eigen value matrix of } J_{R} \)

From (3) and (6), we have
\[
\Delta V = \xi^{*} \Lambda^{-1} \eta \Delta Q
\]
or
\[
\Delta V = \sum_{i} \frac{\eta_{i}}{\lambda_{i}} \xi_{i} \Delta Q
\]
Where \( \xi_{i} \) is the ith column right Eigen vector and \( \eta_{i} \) the ith row left eigenvector of \( J_{R} \).
\( \lambda_{i} \) is the ith eigen value of \( J_{R} \).

The \( i^{th} \) modal reactive power variation is,
\[
\Delta Q_{mi} = K_{i} \xi_{i}
\]
where
\[
K_{i} = \sum_{j} \frac{\xi_{j}}{\lambda_{j}}^{2} - 1
\]
Where
\( \xi_{j} \) is the jth element of \( \xi_{i} \)

The corresponding ith modal voltage variation is
\[
\Delta V_{mi} = [1/\lambda_{i}] \Delta Q_{mi}
\]
In (8), let \( \Delta Q = e_{k} \) where \( e_{k} \) has all its elements zero except the \( k^{th} \) one being 1. Then,
Load bus voltage ($V_{li}$) inequality constraint:

$$V_{li}^{\min} \leq V_{li} \leq V_{li}^{\max}, i \in nl$$  \hspace{1cm} (19)

Switchable reactive power compensations ($Q_{ci}$) inequality constraint:

$$Q_{ci}^{\min} \leq Q_{ci} \leq Q_{ci}^{\max}, i \in nc$$  \hspace{1cm} (20)

Reactive power generation ($Q_{ci}$) inequality constraint:

$$Q_{ci}^{\min} \leq Q_{ci} \leq Q_{ci}^{\max}, i \in ng$$  \hspace{1cm} (21)

Transformers tap setting ($T_{ti}$) inequality constraint:

$$T_{ti}^{\min} \leq T_{ti} \leq T_{ti}^{\max}, i \in nt$$  \hspace{1cm} (22)

Transmission line flow ($S_{li}$) inequality constraint:

$$S_{li}^{\min} \leq S_{li} \leq S_{li}^{\max}, i \in nl$$  \hspace{1cm} (23)

Where, nc, ng and nt are numbers of the switchable reactive power sources, generators and transformers.

IV. BLACK HOLE CONCEPT

A black hole in space is formed when a star of gigantic dimension collapsed. The gravitational supremacy of the black hole is too high and even light cannot get away from it. The gravity is so burly because matter has been compress into a miniature space. Whatever thing that cross the frontier of the black hole will be swallowed by it and disappear. Nothing can get away from its mammoth power. The sphere-shaped frontier of a black hole in space is known as the event horizon. The radius of the event horizon is expressed as the Schwarzschild radius. At this radius, the runaway speed is equivalent to the speed of light, and just the once light passes through, even it cannot escape. Nothing can get away from within the event horizon because nothing can go faster than light. The Schwarzschild radius is computed by the following equation:

$$R = \frac{2GM}{c^2}$$  \hspace{1cm} (24)

Where $G$ is the gravitational constant, $M$ is the mass of the black hole, and $c$ is the speed of light. Whatever thing progress close to the event horizon or crosses the Schwarzschild radius it will be wrapped up into the black hole and everlastingly disappear [18–19]. The continuation of black holes can be discerned by its effect over the objects surrounding it.

V. BLACK HOLE ALGORITHM

The BHA algorithm is a population-based technique that has some common characteristics with other population-based techniques. The same as with other population-based algorithms, a population of candidate solutions to a specified problem is produced and dispersed randomly in the search space. The population-based algorithms develop the created population in the direction of the optimal solution via definite mechanisms. For illustration, in GAs, the developing is done by mutation and crossover procedure. In PSO, this is made by moving the candidate solutions in the region of the search space by means of the best found locations, which are restructured as superior locations are found by the candidates. In the proposed BHA algorithm the development of the population is done by moving all the candidates in the direction of the best candidate in every iteration, that is, the black hole and substitutes those candidates that come into within the range of the black hole by newly produced candidates in the search space. The black hole technique has been used for the first time in solving benchmark functions [20]. The projected BHA algorithm in this paper is more analogous to the natural black hole occurrence and is totally dissimilar from the black hole PSO [20]. In our BHA the most excellent candidate among all the candidates at every iteration is selected as a black hole and all the other candidates form the regular stars. The formation of the black hole is not arbitrary and it is one of the authentic candidates of the population. Then, all the candidates are stimulated towards the black hole based on their existing location and an arbitrary number.

Similar to other population-based algorithms, in the projected black hole algorithm (BHA) an arbitrarily produced population of candidate solutions – the stars – are positioned in the search space of the problem. Following initialization, the fitness values of the population are computed and the most excellent candidate in the population, which has the most excellent fitness value, is selected to be the black hole and the rest form the regular stars. The black hole has the capability to take in the stars that surround it.

After initializing the black hole and stars, the black hole begin to take up the stars surround it and all the stars start moving in the direction of the black hole. The amalgamation of stars by the black hole is formulated as follows:

$$X_i(t + 1) = X_i(t) + rand \times (X_{BH} - X_i(t)) \hspace{1cm} i = 1, 2, \ldots, N$$  \hspace{1cm} (25)

Where $X_i(t)$ and $X_i(t + 1)$ are the locations of the $i^{th}$ star at iterations $t$ and $t + 1$, respectively. $X_{BH}$ is the location of the black hole in the explore space. $rand$ is a arbitrary number in the interval $[0, 1]$. $N$ is the number of stars (candidate solutions).

While moving in the direction of the black hole, a star might reach a position with lower cost than the black hole. In such a case, the black hole moves about to the position of that star and vice versa. Then the BHA algorithm will go on with the black hole in the fresh location and then stars start moving in the direction of this fresh location. In addition, there is a possibility of crossing the event horizon at some stage of moving stars towards the black hole. Every star (candidate solution) that crosses the event horizon of the black hole will
be sucked by the black hole. Every time a candidate (star) expire – it is sucked in by the black hole – an additional candidate solution (star) is born and dispersed arbitrarily in the explore space and starts a fresh search. This is done to maintain the number of candidate solutions constant. Subsequent iteration takes place after all the stars have been moved.

The radius of the event horizon in the black hole algorithm is computed using the following equation:

$$ R = \frac{f_{BH}}{\sum_{i=1}^{N} f_i} $$  \hspace{1cm} (26)

Where $f_{BH}$ is the fitness value of the black hole and $f_i$ is the fitness value of the ith star. $N$ is the number of stars (candidate solutions). When the distance between a candidate solution and the black hole (best candidate) is less than $R$, that candidate is collapsed and a new candidate is created and distributed randomly in the search space. Based on the above explanation the key steps in the BHA algorithm are concise as follows:

- Initialize a population of stars with arbitrary locations in the explore space
- Loop
  - For every star, calculate the objective function
  - Pick the most excellent star that has the most excellent fitness value as the black hole
  - Modify the position of every star according to Eq. (25)
  - If a star attains a position with lower cost than the black hole, then swap their locations
  - If a star crosses the event horizon of the black hole, substitute it with a fresh star in an arbitrary location in the search space
  - If a stop criterion (a highest number of iterations or satisfactorily high-quality fitness) is met, exit the loop
- End loop

VI. SIMULATION RESULTS

The accuracy of the proposed BHA method is demonstrated by testing it on standard IEEE-30 bus system. The IEEE-30 bus system has 6 generator buses, 24 load buses and 41 transmission lines of which four branches are (6-9), (6-10), (4-12) and (28-27) - are with the tap setting transformers. The lower voltage magnitude limits at all buses are 0.95 p.u. and the upper limits are 1.1 for all the PV buses and 1.05 p.u. for all the PQ buses and the reference bus. The simulation results have been presented in Tables 1, 2, 3 & 4. And in the Table 5 shows the proposed algorithm powerfully reduces the real power losses when compared to other given algorithms. The optimal values of the control variables along with the minimum loss obtained are given in Table 1. Corresponding to this control variable setting, it was found that there are no limit violations in any of the state variables.

<table>
<thead>
<tr>
<th>Control variables</th>
<th>Variable setting</th>
</tr>
</thead>
<tbody>
<tr>
<td>V1</td>
<td>1.042</td>
</tr>
<tr>
<td>V2</td>
<td>1.043</td>
</tr>
<tr>
<td>V5</td>
<td>1.042</td>
</tr>
<tr>
<td>V8</td>
<td>1.031</td>
</tr>
<tr>
<td>V11</td>
<td>1.002</td>
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<tr>
<td>V13</td>
<td>1.040</td>
</tr>
<tr>
<td>T11</td>
<td>1.03</td>
</tr>
<tr>
<td>T12</td>
<td>1.01</td>
</tr>
<tr>
<td>T15</td>
<td>1.0</td>
</tr>
<tr>
<td>T36</td>
<td>1.0</td>
</tr>
<tr>
<td>Qc10</td>
<td>4</td>
</tr>
<tr>
<td>Qc12</td>
<td>3</td>
</tr>
<tr>
<td>Qc15</td>
<td>3</td>
</tr>
<tr>
<td>Qc17</td>
<td>0</td>
</tr>
<tr>
<td>Qc20</td>
<td>4</td>
</tr>
<tr>
<td>Qc23</td>
<td>4</td>
</tr>
<tr>
<td>Qc24</td>
<td>2</td>
</tr>
<tr>
<td>Qc29</td>
<td>4</td>
</tr>
<tr>
<td>Real power loss</td>
<td>4.4099</td>
</tr>
<tr>
<td>SVSM</td>
<td>0.2479</td>
</tr>
</tbody>
</table>

ORPD together with voltage stability constraint problem was handled in this case as a multi-objective optimization problem where both power loss and maximum voltage stability margin of the system were optimized simultaneously.

Table II indicates the optimal values of these control variables. Also it is found that there are no limit violations of the state variables. It indicates the voltage stability index has increased from 0.2479 to 0.2488, an advance in the system voltage stability. To determine the voltage security of the
system, contingency analysis was conducted using the control variable setting obtained in case 1 and case 2. The Eigen values equivalents to the four critical contingencies are given in Table 3. From this result it is observed that the Eigen value has been improved considerably for all contingencies in the second case.

### Table II Results of BHA - Voltage Stability Control Reactive Power Dispatch Optimal Control Variables

<table>
<thead>
<tr>
<th>Control Variables</th>
<th>Variable Setting</th>
<th>Variable Setting</th>
</tr>
</thead>
<tbody>
<tr>
<td>V1</td>
<td>1.044</td>
<td></td>
</tr>
<tr>
<td>V2</td>
<td>1.043</td>
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<tr>
<td>V5</td>
<td>1.042</td>
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<tr>
<td>V8</td>
<td>1.032</td>
<td></td>
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<tr>
<td>V11</td>
<td>1.006</td>
<td></td>
</tr>
<tr>
<td>V13</td>
<td>1.033</td>
<td></td>
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<tr>
<td>T11</td>
<td>0.090</td>
<td></td>
</tr>
<tr>
<td>T12</td>
<td>0.090</td>
<td></td>
</tr>
<tr>
<td>T15</td>
<td>0.090</td>
<td></td>
</tr>
<tr>
<td>T36</td>
<td>0.090</td>
<td></td>
</tr>
<tr>
<td>Qc10</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Qc12</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Qc15</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Qc17</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Qc20</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Qc23</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Qc24</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Qc29</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Real power loss</td>
<td>4.9979</td>
<td>0.2488</td>
</tr>
<tr>
<td>SVSM</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table III Voltage Stability Under Contingency State

<table>
<thead>
<tr>
<th>Sl. No</th>
<th>Contingency</th>
<th>ORPD Setting</th>
<th>VSCRPD Setting</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>28-27</td>
<td>0.1410</td>
<td>0.1432</td>
</tr>
<tr>
<td>2</td>
<td>4-12</td>
<td>0.1658</td>
<td>0.1663</td>
</tr>
<tr>
<td>3</td>
<td>1-3</td>
<td>0.1774</td>
<td>0.1772</td>
</tr>
<tr>
<td>4</td>
<td>2-4</td>
<td>0.2032</td>
<td>0.2043</td>
</tr>
</tbody>
</table>

### Table IV Limit Violation Checking Of State Variables

<table>
<thead>
<tr>
<th>State Variables</th>
<th>Limits</th>
<th>ORPD</th>
<th>VSCRPD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1</td>
<td>-20 -15</td>
<td>13.422</td>
<td>-1.3269</td>
</tr>
<tr>
<td>Q2</td>
<td>-20 -61</td>
<td>8.9900</td>
<td>9.8232</td>
</tr>
</tbody>
</table>

### Table V. Comparison of Real Power Loss

<table>
<thead>
<tr>
<th>Method</th>
<th>Minimum loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evolutionary programming[21]</td>
<td>5.0159</td>
</tr>
<tr>
<td>Genetic algorithm[22]</td>
<td>4.665</td>
</tr>
<tr>
<td>Real coded GA with Lindex as SVSM[23]</td>
<td>4.568</td>
</tr>
<tr>
<td>Real coded genetic algorithm[24]</td>
<td>4.5015</td>
</tr>
<tr>
<td>Proposed BHA method</td>
<td>4.4099</td>
</tr>
</tbody>
</table>

### VII. Conclusion

In this paper BHA algorithm is used to solve optimal reactive power dispatch problem by including various generator constraints. The designed method formulate reactive power dispatch problem as a mixed integer non-linear optimization problem. The performance of the designed algorithm has been established well through its voltage stability evaluation by modal analysis and is effective at various instants following system contingencies. The efficiency of the proposed method has been demonstrated on IEEE 30-bus system. Simulation results show that Real power loss has been considerably reduced and voltage profile index has been enhanced.

### VIII. Nomenclature

NB number of buses in the system
N_g number of generating units in the system
t_b tap setting of transformer branch k
$P_d, Q_d$: real power generation at slack bus
$V_i$: voltage magnitude at bus $i$
$P_i, Q_i$: real and reactive powers injected at bus $i$
$P_{gi}, Q_{gi}$: real and reactive power generations at bus $i$
$G_{ij}, B_{ij}$: mutual conductance and susceptance between bus $i$ and $j$
$G_{ii}, B_{ii}$: self conductance and susceptance of bus $i$
$\theta_{ij}$: voltage angle difference between bus $i$ and $j$

REFERENCES


