International Journal of Research in Management, Science & Technology (E-ISSN: 2321-3264)
Vol. 5, No. 1, 75-80, April 17
Available at www.ijrmst.org

Multi-Material Modelling with Geometric References

Vikas Gupta

Mechanical Engg. Deptt.
CDL State Institute of Engg. & Tech. Panniwala Mota
`vikasbrcm@rediffmail.com`

***Abstract*— The need of heterogeneous objects in diverse areas of applications calls for systematic modelling approaches for design, analysis and rapid manufacturing of heterogeneous objects. In this work, the volumetric data set and explicit models are combined to propose a generic and uniform composite model to model, control and modify the material variation. The proposed composite model introduces 'Gradient Reference Approach' which models the material distributions using basic geometric references like points, lines, curves, planes and surfaces. With the developed mathematical model and algorithm, the material composition at any point inside the object can be exactly evaluated at runtime during rapid manufacturing of heterogeneous objects. The approach proposes the contour sub-division algorithm to model heterogeneous objects with complex geometry. The work is extended to model complex material distributions in two and three dimensions. The few examples are discussed to validate the developed computer aided model.**

***Keywords*— Heterogeneous objects; rapid manufacturing; gradient reference approach; modelling; and analysis.**

I. INTRODUCTION

With the advent of heterogeneous objects (HO), diverse properties and advantages of a number of materials can be fully exploited within a single object as per the desired functional requirements. Traditional limitations, which arise due to material incompatibility, like stress concentration, non-uniform thermal expansion, etc. can also be avoided with the fabrication of functional components that involve gradual material variations. Due to their excellent performances and unique features, heterogeneous objects have become very popular in recent years. Conventionally, CAD modelling involves geometric and topology information only, and material information has no major significance. Modelling of heterogeneous objects requires a strategy different than that adopted for homogeneous objects. It requires an advanced modelling approach, which should be capable of providing varying material composition information along with part geometric information. Apart from modelling, the design, analysis, and fabrication of heterogeneous objects also depends upon geometric as well as material distribution information in the object domain.

The issues related to design optimization, analysis and selection of additive manufacturing (AM) process are discussed [24]. Jonathan and Hod [14,15] have discussed about design and analysis of heterogeneous objects with multi material topological optimization. Zhang et al. [29] have optimized the material properties of components made of multi materials. A few rapid prototyping (RP) techniques have the capability to fabricate heterogeneous objects but they also require 3D heterogeneous CAD models for part fabrication. Such models should be capable of providing both geometric and material information. The issues related to slice generation have been discussed for rapid manufacturing of heterogeneous objects along with data retrieval algorithm [3-5]. An efficient CAD model is also required to retrieve geometric and material information for eliminating stair-step effects during slicing. Thus, the development of a suitable computer aided HO model and its utility for downward applications in design, analysis, manufacturing are important issues to realize multi-fold functional components.

A lot of efforts have been made in the past towards heterogeneous object modelling. A number of theoretical representations, material function derivations and optimization techniques have been developed for heterogeneous object modelling. Most of the existing HO representations can be broadly classified into two categories: evaluated and unevaluated modelling. Evaluated models are inexact and represent the discrete objects of interest. Unevaluated models generally do not involve intensive spatial decompositions/subdivisions or discretizations, and theoretically, can provide sufficient fidelity in geometries and material distributions. An approach to model multi-material objects based on R-m sets and R-m classes is proposed [17,18], primarily for application in layered manufacturing. Boolean operators were also defined to facilitate the modelling process. Jackson et al. [12] and Liu et al. [20] have defined a local composition control (LCC) approach to represent heterogeneous objects, where a mesh model is divided into tetrahedrons and different material compositions are evaluated on the nodes of the tetrahedrons, by using Bernstein polynomials. Jackson et al. [12] have proposed the finite element meshes to represent FGM objects in which volume of the object is decomposed into a collection of "linear tetrahedrons". Chiu and Tan [1] have proposed a method based on material tree structure to store different compositions of an object. The material tree was later added to a data file to construct a modified file format suitable for rapid manufacturing. Huang et al. [11] have modeled FGM objects whose materials follow a distribution characterized by the generic Bezier curves. Qian and Dutta [22,23] have proposed a feature-based modelling scheme to represent heterogeneous objects, by defining boundary conditions of a virtual diffusion problem in a solid. The work was further extended by Liu et al. [20], by taking parameterized functions, in terms of distance(s) and functions, using Laplace equation, to smoothly blend various boundary conditions, through which designers could edit geometry and composition simultaneously. Marson and Dutta [21] have developed tensor product solids through heterogeneous solid modelling. Feng et al. [2] have developed a CAD modelling system for the fabrication of functional components with multi material properties. Kou and Tan [16] have proposed a hierarchical representation scheme, for designing and optimizing objects composed of multiple regions, with continuously varying material properties. This approach uses B-rep method to represent geometry and a heterogeneous feature tree to express the material distributions. Zhou and Lu [30] have used distance as the key parameter to distribute the materials in a heterogeneous object. A level-set based variational scheme is proposed [28], which has adapted a variational model as the objective functional to locate any point in the material region of a well-defined gradient or on the boundary edges and surfaces of discontinuities; the set of discontinuities is represented implicitly, using a multiphase level set model. Tsukanov and Shapiro [27] have presented a mesh-free approach based on the generalized Taylor series expansion of a distance field, to model and analyze a heterogeneous object, which satisfies the prescribed material conditions on a finite collection of material features and global constraints. Liu [19] has verified the approaches in commercial software packages, such as Solidworks and Unigraphics. A commercial CAD package independent system is developed by Qian and Dutta [23] to deal with the HO modelling.

Despite the developments as reported in the literature, heterogeneous object modelling has not matured and there are some open issues in existing representations that need to be addressed properly by the research fraternity. As per the best of our understanding, most of the existing computer aided models are insufficient to provide generic and uniform representations of heterogeneous objects. The work on complex geometric and material variations is limited, so, modelling complex HOs needs more focused attention of the research fraternity. Effective computer aided HO models for integration with visualization, analysis and rapid prototyping set-ups are restricted. Thus, developing a computer-aided model for HO is a big challenge as the model is expected to be capable of:

- representing the geometry, topology and material information simultaneously, generically and uniformly;
- providing intuitive modelling tools for not only modelling heterogeneous objects, but also their modification and maintenance;
- representing complex heterogeneous objects, which are generally supposed to have simultaneous complex geometry intricacies and compound material variations.

Based on the above discussion, this work proposes a representation schema for modelling heterogeneous objects that also includes the characteristics of the few available representations in an intuitive and efficient manner. The current work is based on the concept of a composite model that represents heterogeneous objects for downstream applications of modelling, visualization, analysis and fabrication. The composite model includes volumetric data set models i.e. voxel and tetrahedral models, and explicit models. The voxel-based model is efficient in representing geometry of the object; explicit model is best suited for material distribution. Based on these models, this work proposes an approach, called Gradient Reference Approach (GRA), to model material distributions in simple and complex shaped HOs. The basic geometric entities like points, lines, curves, planes and surfaces of the object (or virtually created) are used to create and control the material composition in an object. With this approach, material distribution can be easily modeled and modified at any stage. The material information in 3D object is explicitly defined using gradient references and boundary enclosures. With the proposed mathematical model for material evaluation, the material composition at any point inside the object can be exactly evaluated during runtime. The material properties can also be evaluated for different material compositions across the heterogeneous object domain. The developed approach is flexible and versatile enough to control the material composition at any location in the HO. The approach permits material modelling for complex geometries as well as for complicated3D material distributions.

In this paper, Section II formally introduces the proposed Gradient Reference Approach (GRA) along with various aspects of GRA for heterogeneous object modelling. Section III focuses on controlling and modifying the material distributions through contour sub-division algorithm in the object domain. Finally, the paper is summarized in Section IV.

II. GRADIENT REFERENCE APPROACH (GRA) FOR HETEROGENEOUS OBJECT MODELLING

The gradient reference approach has the capability to model computationally heterogeneous objects [3-9].The approach minimizes various limitations, like non-general, non-uniform, inability to control and modify material distribution, etc. of available HO modelling techniques, and thus, is more flexible to create heterogeneous objects in contrast to already presented methods [1,22,23,27]. The gradient references approach simply requires addressing the following aspects, for modelling of any HO.

- First, a material space, also known as gradient space, is identified for the defined volume of the heterogeneous object in the geometric space of the object. The material or gradient space is a closed volume, which is defined by a set of boundaries known as Boundary Enclosures (BE). Boundary enclosures can, in turn, be defined by actual surfaces of the heterogeneous object and/or user-defined surfaces, termed as real surfaces (RS) and virtual surfaces (VS), respectively.

- Second, mode of material distribution is defined for the desired material gradient in the gradient space. Mode of material distribution defines the way; the material is distributed in the gradient space, and is defined by introducing Gradient References (GR). The gradient references are the basic entities such as point, axis and plane/surface, in or out of geometric space, which originates the gradient space and limit it to the defined boundary enclosures in the object. Some features of the object like planes, surfaces, etc. can also be treated as gradient references and / or boundary enclosures.
- Third, a mathematical model is developed by defining material compositions at the start and end of the gradient region i.e. $M_{c(s)}$ and $M_{c(e)}$ respectively, along with the scheme of material distribution between the end positions of the gradient space [7,8]. Distance is used as a key parameter to define the composition of a point in the gradient space. Distance based material distributed function f(x) is defined to vary the material composition across the gradient space. Gupta et al. [3-10] have developed a mathematical model that represents material distribution gradient function and presented the related work.

The above aspects of gradient reference approach are illustrated in Fig. 1.

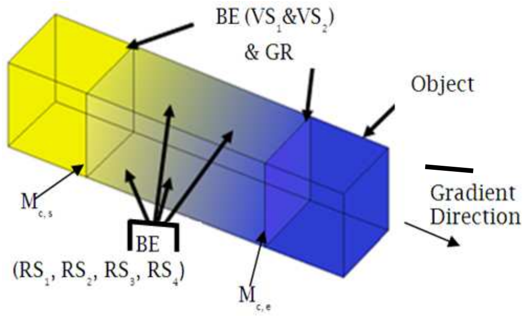
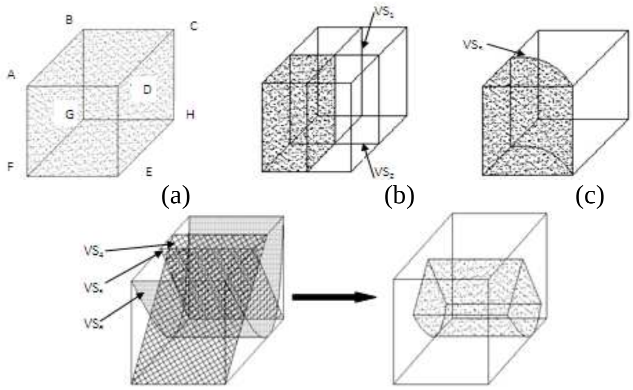


Fig. 1 Aspects of gradient references approach

Gradient space is represented by six boundary enclosures i.e. two virtual surfaces (VS_1 and VS_2) and four real surfaces (RS_1, RS_2, RS_3 and RS_4). Gradient direction along with gradient references (here, VS_1 and VS_2act as GR) is defined. Material composition along the gradient direction, at start and end of the gradient region is $M_{c(s)}$ and $M_{c(e)}$ respectively. Controlling and manipulating the above-defined aspects can realize complex shape heterogeneous objects. In this approach, the basic entities used to define the geometry of an object are normally used as gradient references/boundary enclosures. However, the users can create their own references, independent of geometry of the object, to facilitate local control in HO. The various aspects of gradient reference approach are explained in the following sections.

A. *Boundary Enclosures (BE)*

Boundary enclosures are used to define gradient space in the Geometric space, G^3. The gradient space is defined by a set of boundary enclosures, of distinct shapes. Thus, a complex shaped gradient space can be defined through BE. Fig. 2 presents some examples to demonstrate the modelling of gradient space in the object space.



(a) (b) (c)

(d)

Fig. 2 Modelling of gradient space in the geometric domain

A parallelepiped is modeled to understand the concept of creating gradient spaces. Entire geometric space is considered as gradient space which is bounded by six real BEs i.e. RS_1 = Plane ABCD, RS_2 = Plane DEHC, RS_3 = Plane ADEF, RS_4 = Plane ABGF, RS_5 = Plane BCHG and RS_6 = Plane GHEF, see Fig. 2(a). Gradient space can be modeled by creating user defined planes or surfaces e.g. Fig. 2(b) illustrates the gradient space bounded by real and virtual planes and Fig. 2(c) represents the gradient space bounded by real planes and a virtual curved surface. A complex gradient space can be modeled to fulfill a HO's functional requirements. Fig. 2(d) demonstrates the creation of a complex shaped material space, defined with the help of five BEs i.e. RS_2, RS_4, VS_4, VS_5 and VS_6.

B. *Gradient Reference (GR)*

Gradient references are the basic building blocks for modelling gradient space. With gradient reference, the material distribution can be efficiently mapped to a geometric space. The material composition at any point in the geometric space can be controlled by manipulating gradient references.

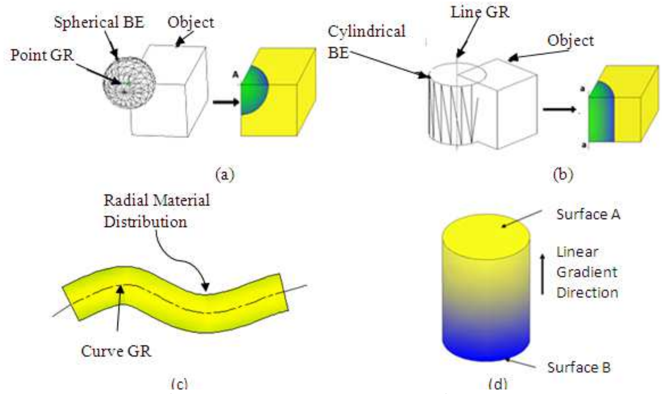


Fig. 3 Gradient references: (a) Point gradient reference; (b) Straight axis gradient reference; (c) Curved axis gradient reference; and (d) Plane gradient references

Gradient references can be classified into the following three categories to model the required material distribution in a given object space, see Fig.3.:

- Point
- Axis: straight/curved
- Plane/ Surface

Point gradient reference provides spherical material distribution as shown in Fig. 3(a). The location of the point, radius of spherical enclosure and the material composition at the center and surface of sphere has to be defined. As the material distribution function in this case is a function of sphere radius, thus, the radius of the sphere controls the material distribution in the gradient space. Axis gradient reference generates material distributions around the straight/curve axis in the radial direction. Radius is a key parameter of material distribution function to distribute the material in the radial direction. Plane/surface gradient reference offers planar material distribution along a gradient direction. The material composition is controlled by manipulating distance between selected/created planes/surfaces. The planes/surfaces associated with an object can be considered as a GR or the user can define/create his/her own planes to distribute the material in a gradient space. Fig. 3(d) describes the concept of linear material distribution through surface gradient references.

Gradient references are also classified as real and virtual gradient references. Any entity i.e. point, axis or plane/surface, which is a basic building block of a 3D heterogeneous object, is termed as ‘real gradient reference’ (RGR). Any user defined geometric entity beyond the entities used for geometric modelling is termed as ‘virtual gradient reference’ (VGR). The gradient references shown in Fig. 3, i.e. vertices of a cuboid, axis of a flexible shaft and top and bottom surfaces of a cylinder, are examples of real gradient references. Examples of virtual gradient references are illustrated in Fig. 4.

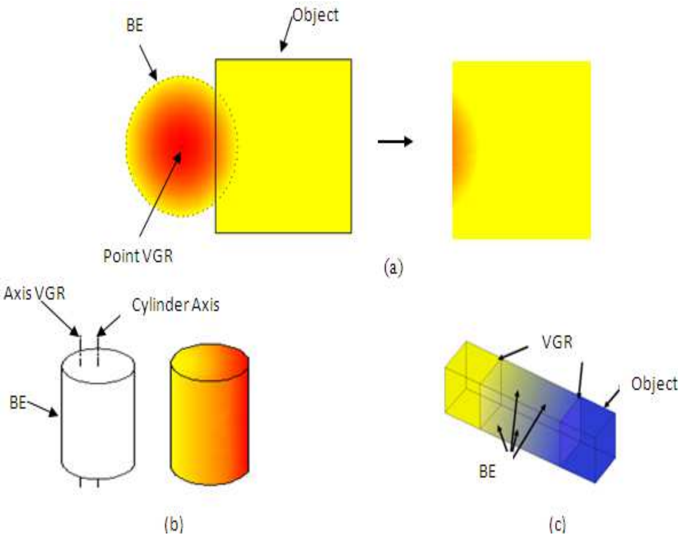


Fig. 4 Virtual gradient references: (a) Point VGR, (b) Axis VGR; and (c) Plane VGR

C. *Mathematical model for Gradient Reference Approach*

Gupta et al. [5,6] have developed a mathematical model to distribute the materials in abounded gradient space. Each point in a heterogeneous object domain is mathematically defined for geometric and material information. Equations are presented to define materials composition with respective volume fraction at each point in the object domain. The resultant properties at each point inside a heterogeneous object are also defined as a function of volume fraction and respective properties. The variations in material distribution and the desired properties are calibrated with the help of distance based material distribution functions. These functions represent the scheme of material distribution in a gradient space i.e. linear or nonlinear, which can be used to control the composition variation in the heterogeneous object domain. Designer or user can define own functions as per the functional requirement in the direction of material gradient.

III. CONTOUR SUB-DIVISION ALGORITHM FOR ONE-DIMENSIONAL MATERIAL DISTRIBUTION

A contour sub-division algorithm is proposed to interpolate one-dimensional (1D) material distribution offered by GRA [4]. In order to distribute the materials in such cases i.e. involving 1D material variations, two-dimensional (2D) gradient region of the object is considered. End contours (i.e. BE and GR) in the direction of gradient are identified. The gradient region between end-contours is divided into a number of sub-regions. Material composition in each sub-region is constant but varies from one sub-region to another in the gradient direction. Material composition for each sub-region is achieved through distance based distribution functions.

A. *Associative and non-associative material distributions*

As discussed earlier, three types of gradient references i.e. point, line/curve and surface/plane are defined for modelling spherical, radial and linear material distributions, respectively. These distributions are bounded by a set of boundary enclosures, which defines the gradient space in HO domain. The concept of GRs and BEs offers freeform material modelling in which the shape of BEs has no effect on the mode of material distribution, see Fig. 5.

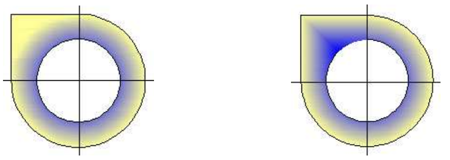

(a) (b)

Fig. 5 Relationship between GRs/BEs: (a) Non-associative; and (b) Associative

The GRs and BEs are independent of each other, which indicate non-associative relation between them, see Fig. 5(a). The examples discussed in previous sections demonstrate the non-associative relation. On the other hand, a number of applications require that the material distributions may adopt the shape of BE, especially real Bes. For example, gradient material modelling at the surface of the objects provides an alternate to replace the objects with irregular surface hardening treatments. In such cases, the pattern of material distribution depends upon shape of gradient space, thus, predicts associative relation between GRs and BEs, see Fig. 5(b).

The material modelling for associated relations is a tedious problem. The contour sub-division algorithm developed in the course of this work has the ability to model materials for both types of distributions. The associative relations classify a set of involved BEs and GRs in the following three categories, as shown in Fig. 6.

- Similar shape and parallel/concentric
- Similar shape and non-parallel/non-concentric
- Dissimilar shape and non-parallel/non-concentric

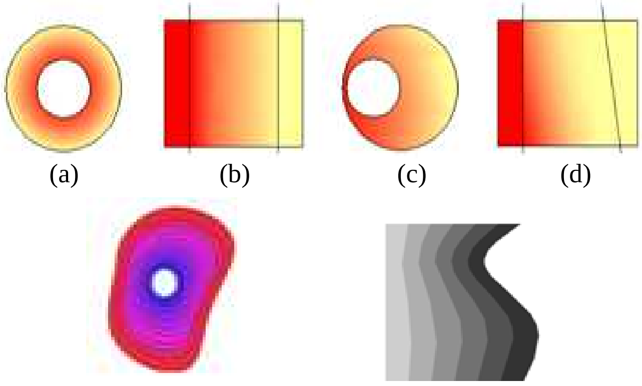

(a) (b) (c) (d)
(e) (f)

Fig. 6 Associative material distributions: (a) similar shape and concentric, (b) similar shape and parallel, (c) similar shape and non-concentric, (d) similar shape and non-parallel, (e) dissimilar shape and non-concentric, and (f) dissimilar shape and non-parallel

Gradient reference approach with contour sub-division algorithm offers the capability to model material distributions in complex shape heterogeneous objects. However, the methodology to model complex material distributions i.e. 2D and 3D is presented in following sections.

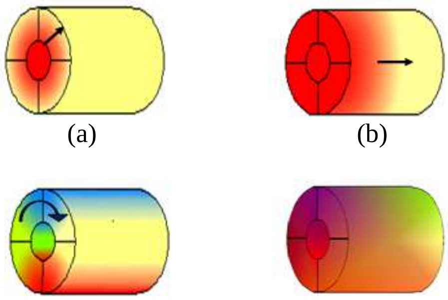

(a) (b)
(c) (d)

Fig. 7. 3D Material distribution in a cylindrical shell: (a) Radial material distribution; (b) Linear material distribution; (c) Angular material distribution; and (d) 3D combined distribution.

An example of 3D material distribution in a cylindrical shell is shown in Fig. 7. Material distributions in three directions i.e. radial, linear and angular are combined to get complex 3D distribution in cylindrical shell. Fig. 7(a) illustrates the radial material distribution while Fig. 7(b) demonstrates the linear distribution in the cylindrical shell. Third distribution in angular direction along the axis is shown in Fig. 7(c). 3D view of heterogeneous cylinder shell is presented in Fig. 7(d).

IV. CONCLUSIONS

In this work, a gradient reference approach is evolved to model material gradients in heterogeneous objects. The approach has three main features i.e. gradient references, boundary enclosures and material composition functions; and they are developed, here, for modelling materials in HO domain. The features are manipulated to control and modify the material distributions. The approach organizes the material variation shape dependency relationships between gradient references and boundary enclosures. With the proposed contour sub-division material distribution algorithm, the material distribution for any kind of HO can be exactly processed during runtime. The methodology for two and three dimensional material distribution is developed for modelling complex material distributions.

Compared with the existing heterogeneous object modelling approaches, GRA offers a novel way to model material distributions through basic geometric entities. The approach is flexible to model material distributions which can be manipulated through geometric features/parameters. Both linear and non-linear material gradients can be effectively modeled, thus, making the approach general in nature. Other features like ability to control and modify distributions and capability to model materials for complex geometry and complex distributions make this approach more generic and uniform. Successful implementation of the developed representation has been demonstrated with a few examples.

V. REFERENCES

[1] Chiu WK, Tan ST. Multiple material objects: from CAD representation to data format for rapid prototyping. Computer-Aided Design 2000; 32:707–17. DOI:10.1016/S0010-4485(00)00046-4

[2] Feng Z, Chen K, Feng X. Development of a CAD modelling system for components made of multi heterogeneous materials. Materials and Design 2005; 26 :113.

[3] Gupta V. Algorithm and Data Structure for Realization of Heterogeneous Objects, , International Journal of Research in Mgt., Sci. & Tech. 2016; 4(2): 13-17.

[4] Gupta V. CAD Paradigm for Modeling, Analyses, Visualization and Rapid Manufacturing of Heterogeneous Objects, International Journal of Research in Mgt., Sci. & Tech. 2016; 4(2): 94-101.

[5] Gupta V, Bajpai VK, Tandon P. Slice generation and data retrieval algorithm for rapid manufacturing of heterogeneous objects. Computer Aided Design & Applications 2014;11(3):255-262. DOI: 10.1080/16864360.2014.863483

[6] Gupta V, Kasana KS, Tandon P. Reference based geometric modelling for heterogeneous objects. Computer Aided Design & Applications 2012; 9(2): 155-165.

[7] Gupta V, Kasana KS, Tandon P. CAD modelling, algorithm design, and system structure for rapid prototyping of heterogeneous objects. International Journal of Computer Applications in Engineering, Technology and Sciences 2010; 2(2): 299-303.

[8] Gupta V, Kasana KS, Tandon P. Computer aided design modelling for heterogeneous objects, International Journal of Computer Science Issues 2010; 7(5): 31-38.

[9] Gupta V, Tandon P. Heterogeneous composition adaption with material convolution control features, Journal of Computing & information Science in Engg. (ASME USA) June 2017; 1. DOI: 10.1115/1.403474.

[10] Gupta V, Tandon P. Heterogeneous Object Modeling with Material Convolution Surfaces, Computer Aided Design (Elsevier, UK) May 2015; 62: 236-247.

[11] Huang J, Fadel GM, Blouin VY, Grujicic M. Bi-objective optimization design of functionally gradient materials. Materials & Design 2002;23: 657.

[12] Jackson TR, Liu H, Patrikalakis NM, Sachs EM, Cima MJ. Modelling and designing functionally graded material components for fabrication with local composition control. Material & Design 1999; 20(2-3):63.

[13] Jackson TR. Analysis of functionally graded material object representation methods. Ph.D. thesis, Massachusetts Institute of Technology (MIT), Cambridge, MA 2000.

[14] Jonathan DH, Hod L. Design and analysis of digital materials for physical 3D voxel printing. Rapid Prototyping Journal 2009: 15(2): 137–149. DOI:10.1108/13552540910943441

[15] Jonathan DH, Hod L. Multi Material Topological Optimization of Structures and Mechanisms. GECCO'09, Montréal Québec, Canada 2009. DOI:10.1145/1569901.1570105

[16] Kou XY, Tan ST. A hierarchical representation for heterogeneous object modelling. Computer-Aided Design 2004: 37(3): 307. DOI:10.1016/j.cad.2004.03.006

[17] Kumar V, Dutta D. An approach to modelling and representation of heterogeneous objects. Journal of Mechanical Design 1997; 120: 659–67. DOI:10.1115/1.2829329

[18] Kumar V, Dutta D. An approach to modelling multi-material objects. Proceedings of the Fourth ACM Symposium on Solid Modelling and Applications 1997: 336–45. DOI:10.1145/267734.267812

[19] Liu H. Algorithms for design and interrogation of functionally graded material solids. Master's thesis, Massachusetts Institute of Technology, Cambridge, MA 2000. http://czms.mit.edu/cho/3dp/onr/report/0700/hl-thesis.pdf

[20] Liu H, Maekawa T, Patrikalakis NM, Sachs EM, Cho W. Methods for feature-based design of heterogeneous solids. Computer-Aided Design 2004; 36(11): 41–59. http://www.mit.edu/~tdp/info-flow/publications/CADPaper.pdf

[21] Marsan A, Dutta D. On the application of tensor product solids in heterogeneous solid modelling. Proceedings of 1998 ASME Design Engineering Conferences, Atlanta, Georgia 1998: 1–9.

[22] Qian X, Dutta D. Feature-based design for heterogeneous objects. Computer-Aided Design 2004; 36(12): 1263–78. DOI:10.1016/j.cad.2004.01.012

[23] Qian X, Dutta D. Physics-based modelling for heterogeneous object. Trans ASME 2003; 125: 416-427. DOI:10.1115/1.1582877

[24] Rao RV, Padmanabhan KK. Rapid prototyping process selection using graph theory and matrix approach. Journal of Materials Processing Technology 2007; 194: 81–88. DOI:10.1016/j.jmatprotec.2007.04.003

[25] Tandon P, Kant A. A CAD system for heterogeneous solid modelling. International Journal of Computer Applications in Technology 2005; 24(4): 226-235. DOI:10.1504/IJCAT.2005.008269

[26] Tandon P, Kant A. A modelling and manufacturing strategy for the heterogeneous solids. International Journal of Manufacturing Technology and Management 2005; 6(5): 485-500. DOI:10.1504/IJMTM.2004.005674

[27] Tsukanov I, Shapiro V. Mesh free modelling and analysis of physical fields in heterogeneous media. Adv Comput Math 2003; 23(1-2): 95-124. DOI:10.1007/s10444-004-1835-3

[28] Wang MY, Wang XM. A level-set based variational method for design and optimization of heterogeneous objects. Computer-Aided Design 2005; 37(3): 321-337. DOI:10.1016/j.cad.2004.03.007

[29] Zhang XJ, Chen KZ, Feng XA. Optimization of material properties needed for material design of components made of multi-heterogeneous materials. Materials & Design 2004; 25: 369.

[30] Zhou H, Liu Z, Lu B. Heterogeneous object modelling based on multi-color distance field. Materials and Design 2009; 30: 939.